Optimal choice of FEC parameters to protect against queueing losses in wireless networks

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***Abstract*— This paper proposes algorithms to determine an optimal choice of the FEC parameters (n, k) to mitigate against the effects of packet loss due to buffer overflow at a wireless base station on multimedia traffic. To solve this problem, we develop analytic models of the considered network scenarios that take into account traffic the arrival rates, the channel loss characteristics, and the size of the buffer at the wireless base station. We develop an iterative procedure to compute the packet loss probabilities after FEC recovery and verify the analytical results via simulation.**

I. Introduction

The delivery of real-time multimedia traffic over wireless networks will be a significant application in future wireless networks. The perceptual quality of service received by the user suffers from packet loss, delay, and delay jitter. Packet loss in wireless networks is due to two causes:

1. Packets might get lost or arrive corrupted at the receiver when they are transported over the lossy medium air. This can be the result of a low signal to noise ratio or a collision with packets sent from neighboring base stations.
2. Packets are dropped from the queue of the wireless access point (AP) if the queue is full.

Throughout this paper, we will refer to packet losses due to the reasons 1 and 2 as *losses of type 1* and *2* respectively.

Forward Error Correction (FEC) codes have been extensively researched as a method to improve QoS [1], [4], [11]. They have proved to be very efficient if packet losses are not too bursty. A FEC block code is characterized by the parameter set (n, k) : Each code takes a codeword of k data packets and generates $n - k$ additional parity packets and transmits all n packets over the network. If at least k out of these n packets are received at the receiver, then all k multimedia packets are recovered. Otherwise, only the received multimedia packets are recovered. In the sequel, we denote k multimedia packets and the $(n - k)$ FEC packets protecting them as a *multimedia sequence*. Adaptive FEC protections schemes that dynamically change the FEC parameters (n, k) in view of rapidly changing network conditions have been researched in [5], [6].

The papers listed above do not distinguish between packet losses of type 1 and 2. Also, the effectiveness of a new FEC parameter set is calculated using the current loss probability of the considered traffic stream, i.e., it is assumed that the loss probability of the channel is not influenced by a change in the sending rate of the multimedia traffic due to a change in the set of FEC parameters. For losses of type 1 this assumption is roughly correct, if one assumes that the channel has enough bandwidth to send the original multimedia packets as well as the new FEC packets at the same bit rate it was operating before changing the actual FEC parameters (n, k). In contrast, for packet losses of type 2, a change of the FEC parameters leads to a change in the overall sending rate of the multimedia traffic that in turn leads to a different occupancy of the AP queue and thus influences the frequency at which losses of type 2 occur. Consequently, we see that the algorithms described in previous papers only predict the effectiveness of FEC codes correctly for losses of type 1, but not of type 2.

The effectiveness of FEC parameters in view of losses of type 2 was investigated in [2],[7], [8], and [12]. However, in these papers, the sending pattern of the multimedia traffic is modeled as a Poisson traffic, which does not model accurately the common practice of sending multimedia traffic over communication networks at a constant bit rate.

In this paper we consider the problem of determining the efficiency of FEC parameters (n, k) in view of losses of type 2. In contrast to [8], we use a discrete time model that allows us to both exactly model the dynamics of a constant bit rate sending process and derive a shorter mathematical derivation than in [8]. In our approach, the current packet loss probability is not used explicitly as an input parameter for the algorithm to determine the FEC parameters. Instead, we develop a model of the network that takes explicitly into account the arrival patterns of all traffic streams in the network, the size of the buffer at the access point, and the reliability of the wireless channel. We note that in [2],[7], and [12] only one traffic stream is considered, whereas we distinguish between a primary multimedia traffic and competing traffic streams.

Technically, we first develop and solve a Markov based steady state model for the queue occupancy using a singular value decomposition. Then, we develop an iterative procedure to calculate the loss probability of multimedia packets after FEC recovery in dependence of the FEC parameters (n, k).

Because this procedure to calculate the effectiveness of FEC codes is computationally too complex for a real-time application, we also propose an approximation algorithm that allows to determine an optimal choice of the parameters (n, k) under the real-time constraints of a typical multimedia application. The reduction of the complexity is achieved by considering only a subset of the steady state model without significantly compromising the accuracy of the calculations.

We describe the considered network model in section II. In section III, we specify a sending scheme for multimedia CBR traffic protected by an FEC code. In section IV, we develop an algorithm to determine the loss probability for multimedia packet after FEC recovery. We describe an approximation algorithm in section V. We provide numerical evidence for the correctness of the algorithms developed in section VI and we conclude in section VII.

II. Network model and terminology

We consider a network (fig. 1) that consists of a multimedia sender and a multimedia receiver, a number of competing senders and a number of competing receivers, and a wireless access point (AP). The multimedia sender sends multimedia packets and the competing senders send unspecified packets over wired connections to the AP. The AP forwards all packets to the multimedia or competing receivers. For simplicity, we assume that all packets are of fixed, equal size.

Packets that arrive at the AP from the senders are either directly sent to the wireless interface of the AP or are queued in a buffer within the AP. The AP-queue has a maximum length T which corresponds to the maximum number of packets that can be buffered at the AP. Any packets that arrive when the queue is full are dropped from the queue. In this paper, we only consider AP-queues that apply a FIFO queueing discipline. The first packet of a queue is sent over the wireless channel if the channel is available. The wireless technology might implement its own QoS technologies as retransmission or MAC layer FEC. Throughout this paper,

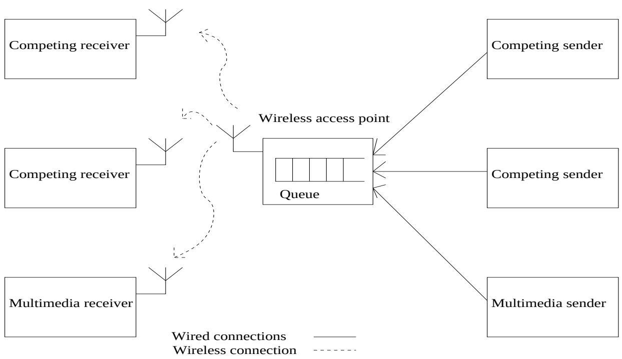

Fig. 1. Network model

we assume a discrete, slotted time model. For simplicity, we assume that both the arrival of a packet to the AP from a sender and the successful departure of a packet from the AP over the wireless channel take one time slot. We assume that a multimedia packet is sent every m time slots. The sending scheme of the FEC packets is specified in section III. We assume that the arrival rate of unspecified competing traffic is distributed according to a Bernoulli distribution with a parameter $p_C < 1$ and $q_C = 1 - p_C$. We define p_D as the probability that in a time slot a packet is sent successfully from the AP queue over the wireless channel and $q_D = 1 - p_D$. p_D presents the rate at which packets leave the AP buffer. It does not include the rate at which retransmission or MAC layer FEC packets are sent over the wireless channel if any of these technologies is implemented in the wireless network. For the analysis, we split a time slot in three phases: the packet arrival phase, the packet dropping phase, and the packet sending phase which occur in the same chronological order:
1. Packet arrival phase: Packets are sent from the multimedia sender to the AP according to a predefined sending scheme explained in the section III. Competing packets are sent from the competing senders with probability p_C to the AP. If packets are sent from both the multimedia and senders, we assume that each of the two packets arrives before the other packet with equal probability $\frac{1}{2}$. The queue length is increased by the number of arriving packets $a \in \{0, 1, 2\}$.
2. Packet dropping phase: If, after the packet arrival phase, there are $T + d$, $d \in \{1, 2\}$ packets in the AP-queue, the last d packets are dropped from the queue.
3. Packet sending phase: With probability p_D the first packet of the AP-queue is sent over the wireless channel. If the queue is empty, no packet is sent. The queue length is decreased by the number of departing packets $d \in \{0, 1\}$.

The scope of this paper is to calculate the loss probability after FEC recovery for given parameters m, p_C, p_D, T, n, and k.

III. Sending scheme for CBR traffic with FEC protection

In this section, we define a sending scheme for CBR multimedia traffic that is protected using an FEC code. We say that a CBR multimedia stream is sent with a periodicity of $m \geq 2$ if a multimedia packet is sent every m time slots whereas no multimedia packet is sent during the $m - 1$ intermediate time slots. We further assume the deployment of FEC codes with a parameter set (n, k), i.e, k- original multimedia packets and $n - k$ additional FEC packets are sent to the queue from the multimedia sender. Assuming a sending periodicity m for the k multimedia packets, the $n - k$ FEC packets are sent as follows: After the last of the k multimedia packets of a *current* multimedia sequence has been sent, the $(n - k)$ FEC packets of this multimedia sequence are sent in the next $n - k$ free time slots, i.e., the time slots not used by the k - multimedia packets of the *next* multimedia sequence (see an example in fig. 2). To guarantee the feasibility of this sending scheme, we only allow parameter sets (n, k) such that the number of FEC packets $n - k$ is less then than the number of free time slots between the next k multimedia packets, i.e.,

$$n - k \quad \leq \quad k(m - 1). \tag{1}$$

Fig. 2. Multimedia and FEC packet sending scheme

IV. Packet loss probability after FEC recovery

A. Analytical approach

For the calculation of the loss probability, we assume that the dropping behavior of the first packet of any multimedia sequence which is sent to the AP is independent of the dropping behavior of any previous packet. For any of the remaining $n - 1$ packets of a multimedia sequence, we will calculate its dropping probability in dependence of the dropping behavior of the multimedia or FEC packet which was most recently sent to the AP. This procedure is motivated by the fact that after a multimedia or FEC packet is lost, an analysis of our model in section II shows the queue length is at least $T-1$. This high occupancy of the queue influences the queue occupancy of the time slot when the next multimedia or FEC arrives which in turn influences the loss probability of the next arriving multimedia or FEC packet.

B. Transition matrix

The calculation of the loss probability after FEC recovery will require the calculation of the transition matrix P^t whose entries are the transition probabilities between the different states of the queue occupancy in consecutive time slots. We denote the transition matrix as P^t and define as $P^t_{a,b}$, $0 \leq a, b \leq T$, the probability that the queue length changes from a to b between consecutive time slots where the queue length is measured after the packet sending phase. For any *fixed* time slot, we define a quantity p_A as the probability that either a multimedia or an FEC packet arrives in the *next* time slot. As we use a deterministic sending scheme for the sending of the multimedia and FEC packets, there is $p_A \in \{0, 1\}$ for any time slot. Throughout this paper, we denote the transition matrix P^t as $F = P^t$ if $p_A = 1$ and as $G = P^t$ if $p_A = 0$. We obtain from the model described in section II and fig. 3,

$$
\begin{aligned}
P^t_{0,0} &= q_A q_C + q_A p_C p_D + p_A q_C p_D, \\
P^t_{k-2,k} &= p_A p_C q_D, \; 2 \leq k \leq T, \\
P^t_{k-1,k} &= p_A p_C p_D + p_A q_C q_D + q_A p_C q_D, \\
&\quad 1 \leq k \leq T-1, \\
P^t_{k,k} &= q_A q_C q_D + q_A p_C p_D + p_A q_C p_D \\
&\quad 1 \leq k \leq T-2, \\
P^t_{k+1,k} &= q_A q_C p_D, \; 0 \leq k \leq T-1, \\
P^t_{T-1,T-1} &= q_A q_C q_D + q_A p_C p_D + p_A q_C p_D + p_A p_C p_D, \\
P^t_{T-1,T} &= p_A q_C q_D + q_A p_C q_D + p_A p_C q_D, \\
P^t_{T,T-1} &= p_D, \\
P^t_{T,T} &= q_D, \\
P^t_{a,b} &= 0 \; for \; all \; other \; a \; and \; b.
\end{aligned}
$$

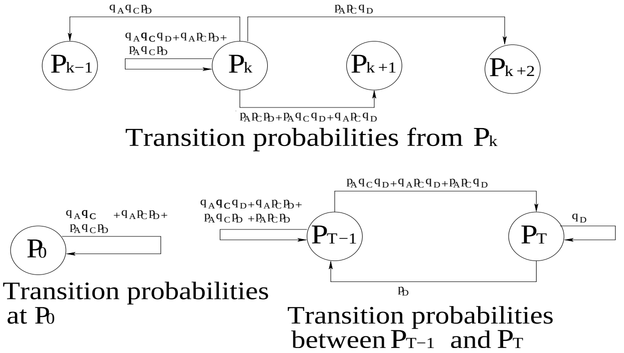

Fig. 3. The transition probabilities for the AP queue occupancy. (P_k denotes the probability that the queue occupancy is equal to k.)

C. Steady state vector of queue occupancy

The calculation of the loss probability after FEC recovery will require knowledge of the steady state vector $S = (S_k)_{0 \leq k \leq T}$ where S_k expresses the probability that the AP queue contains k packets at the end of the sending phase of the time slot before the first packet of a multimedia sequence arrives. We now calculate the vector S. We set $w = \lfloor ((m/(m-1)) * (n-k)) \rfloor$. Then, by the definition of the sending scheme in section III, a multimedia or FEC packet is sent during each of the first w of the considered $k * m$ time slots. In particular, all the $n-k$ FEC packets of the previous multimedia sequence and - if $m \leq w$ - some of the multimedia packets of the current multimedia sequence will be sent during the first w time slots. (We note that by (1), there is $w \leq k*m$.)

We set $u \equiv w(mod\, m)$, $0 \leq u \leq m-1$, and $q = m - u$. Then, $q-1$ equals the number of time slots between the w-th time slot and the next time slot in which a multimedia or FEC packet is sent. Further, we define p via $k*m - w - f*q = mp$ and set $f = min(u, 1)$. Now, we can write the steady state equations for the vector S. We distinguish the cases $m-1 < w$ (fig. 2) and $m-1 > w$ (We note that due to (1) there is always $w \not\equiv m-1 (mod\, m)$. Then, according to the definition of the sending process in section III:

$$
\begin{aligned}
S &= \left(F^{w-m+1}(G^{q-1}F)^f (G^{m-1}F)^p F^{m-1}\right)^T S, \\
&\quad \text{if } m-1 < w, \\
S &= \left(F(G^{m-1}F)^p F^w G^{(q-1)}\right)^T S, \quad \text{if } m-1 > w.
\end{aligned}
$$

We solve these equations for S via a singular value decomposition and use $\sum_i S_i = 1$.

D. Loss probability after FEC recovery

The calculation of the loss probability after FEC recovery is performed in five steps. In the first step, we calculate the conditional probability $R^{j-1}_{g,h}$, $2 \leq j \leq n$ that the queue is of length h at the end of the time slot before the j-th multimedia packet arrives given that the queue was of length g at the end of the time slot when the $j-1$-th multimedia or FEC packet arrived. From the definitions of w, u, and q in section IV-C, and noting that $q \neq 1$ as always $w \not\equiv m-1(mod\, m)$, we find

$$
R^{j-1}_{g,h} = \left\{
\begin{array}{ll}
(F^{m-1})_{g,h}, & 2 \leq j \leq \lfloor \frac{w}{m} \rfloor, \\
(F^u G^{q-1})_{g,h}, & j = \lfloor \frac{w}{m} \rfloor + 1, \; u \geq 1, \; q \geq 2, \\
(G^{m-1})_{g,h}, & j = \lfloor \frac{w}{m} \rfloor + 1, \; u = 0, \\
(G^{m-1})_{g,h}, & \lfloor \frac{w}{m} \rfloor + 1 < j \leq k, \\
\delta_{g,h}, & k+1 \leq j \leq n,
\end{array}
\right\}
$$

where $\delta_{g,h} = 1$ for $g = h$ and $\delta_{g,h} = 0$ else. We recall that the term $R^{j-1}_{g,h}$ is only defined for $2 \leq j \leq n$. Thus, if $j \notin [2, n]$ in any of the conditions of the right side of definition of $R^{j-1}_{g,h}$, then the case described by this condition does not occur.

In the second step, we consider the conditional probability that an arriving multimedia or FEC packet of a multimedia sequence is *not dropped* if the AP queue is of length h at the end of the time slot before the considered multimedia or FEC packet arrives. The arriving multimedia packet is not lost if $h \leq T-2$ or if $h = T-1$, $k = T-2$. If $h = k = T-1$, then the arriving packet is not dropped if either no competing packet arrives and a packet is sent from the queue - with probability $q_C p_D$ - or when a competing packet arrives after the multimedia packet and a packet is sent from the queue - with probability $\frac{1}{2}p_C p_D$. If $h = T-1$, $k = T$, then the arriving packet is not dropped if either no competing packet arrives and no packet is sent from the queue - with probability $q_C q_D$ - or when a competing packet arrives after the multimedia packet and no packet is sent from the queue - with probability $\frac{1}{2}p_C q_D$. If $h = T$, the arriving multimedia packet is dropped.

Thus, given the arrival of a multimedia or FEC packet in the current time slot and given a queue occupancy of h at the end of the previous time slot, the conditional probability that the queue length is equal to k at the end of the sending phase of the current time slot and that the arriving multimedia or FEC packet is *not dropped,* is equal to $E_{h,k}$ where

$$E_{h,k} = \left\{ \begin{array}{ll} F_{h,k}, & h \le T-2, \ or \\ & h = T-1, \ k \le T-2, \\ q_C p_D + \frac{1}{2}p_C p_D, & h = k = T-1, \\ q_C q_D + \frac{1}{2}p_C q_D, & h = T-1, \ k = T, \\ 0, & all\ other\ h\ and\ k. \end{array} \right\}.$$

Similarly, given the arrival of a multimedia or FEC packet in the current time slot and given a queue occupancy of h at the end of the previous time slot, the conditional probability that the queue length is equal to k at the end of the sending phase of the current time slot and that the arriving multimedia or FEC packet is *dropped,* is equal to $E^*_{h,k}$ where

$$E^*_{h,k} = \left\{ \begin{array}{ll} \frac{1}{2}p_C p_D, & h = k = T-1, \\ \frac{1}{2}p_C q_D, & h = T-1, \ k = T, \\ p_D, & h = T, \ k = T-1, \\ q_D, & h = k = T, \\ 0, & all\ other\ h\ and\ k. \end{array} \right\}.$$

In the third step, we calculate the conditional probability $A_{k,g,j}$ that given a queue length g at the end of the time slot in which the $j-1$-th multimedia or FEC packet of a multimedia sequence arrives, the queue is of length k at the end of the sending phase of the time slot in which the j-th multimedia or FEC packet of the multimedia sequence arrives and the j-th multimedia or FEC packet *is not dropped* during the dropping phase. We obtain

$$A_{g,k,j} = \sum_{h=0}^{T} R^{j-1}_{g,h} E_{h,k}, \ 0 \le g,k \le T, \ 2 \le j \le n.$$

Analogously, we calculate the conditional probability $B_{k,g,j}$ that given a queue length g at the end of time slot in which the $j-1$-th multimedia or FEC packet of a multimedia sequence arrives, the queue is of length k at the end of the sending phase of the time slot in which the j-th multimedia or FEC packet of the multimedia sequence arrives and the j-th multimedia or FEC packet *is dropped* during the dropping phase. Thus,

$$B_{g,k,j} = \sum_{h=0}^{T} R^{j-1}_{g,h} E^*_{h,k}, \ 0 \le g,k \le T, \ 2 \le j \le n.$$

In the fourth step, we determine $T(a,b,d)$, which we define as the probability to lose $0 \le b \le a$ packets out of the first a, $1 \le a \le n$ multimedia or FEC packets and to have a queue occupancy d at the end of the sending phase of the time slot in which the a-th packet arrives. For $a = 1$ the $T(a,\cdot,\cdot)$ are functions of the steady state vector S, whereas for $a > 1$, we define the $T(a,\cdot,\cdot)$ recursively using the expressions $A_{g,k,j}$ and $B_{g,k,j}$. We obtain

$$\begin{array}{rcl} T(1,0,0) &=& S_0 q_C p_D, \\ T(1,0,1) &=& S_0(q_C q_D + p_C p_D) + S_1 q_C p_D, \\ T(1,0,k) &=& S_{k-2} p_C q_D + S_{k-1}(q_C q_D \\ &+& p_C p_D) + S_k q_C p_D, \qquad 2 \le k \le T-2, \\ T(1,0,T-1) &=& S_{T-3} p_C q_D + S_{T-2}(q_C q_D + p_C p_D) \\ &+& S_{T-1}(q_C p_D + \frac{1}{2}p_C p_D), \\ T(1,0,T) &=& S_{T-2} p_C q_D + S_{T-1}(q_C q_D + \frac{1}{2}p_C q_D), \\ T(1,1,k) &=& 0, \qquad 0 \le k \le T-2, \\ T(1,1,T-1) &=& S_{T-1}\frac{1}{2}p_C p_D + S_T p_D, \\ T(1,1,T) &=& S_{T-1}\frac{1}{2}p_C q_D + S_T q_D, \\ T(a,0,d) &=& \sum_{c=0}^{T} T(a-1,0,c) A_{c,d,a}, \ 2 \le a \le n, \\ T(a,b,d) &=& \sum_{c=0}^{T} T(a-1,b,c) A_{c,d,a}, \\ &+& \sum_{c=0}^{T} T(a-1,b-1,c) B_{c,d,a}, \\ && 2 \le a \le n, \ 1 \le b \le a-1, \\ T(a,a,d) &=& \sum_{c=0}^{T} T(a-1,a-1,c) B_{c,d,a}, \ 2 \le a \le n. \end{array}$$

In the fifth step, we calculate $S(a,b,d,v)$, $1 \le a \le n-k$, $0 \le b \le a$, $0 \le v \le k$, which we define as the probability to lose v out of k multimedia packets, to lose b FEC packets out of the first a FEC packets and to have a queue length d at the end of the sending phase of the time slot in which the a-th FEC packet arrives. We calculate the expressions $S(1,\cdot,\cdot,\cdot)$ as functions of the $T(k,\cdot\cdot)$ and the $A_{g,k,j}$ and $B_{g,k,j}$, whereas for $a > 1$ we define $S(a,\cdot,\cdot,\cdot)$ recursively from $S(a-1,\cdot,\cdot,\cdot)$, $A_{g,k,j}$ and $B_{g,k,j}$. We find

$$\begin{array}{rcl} S(1,0,d,v) &=& \sum_{c=0}^{T} T(k,v,c) A(c,d,k+1), \\ S(1,1,d,v) &=& \sum_{c=0}^{T} T(k,v,c) B(c,d,k+1), \\ S(a,0,d,v) &=& \sum_{c=0}^{T} S(a-1,0,c,v) A(c,d,a+k), \\ && 2 \le a \le n-k, \\ S(a,b,d,v) &=& \sum_{c=0}^{T} S(a-1,b,c,v) A(c,d,a+k), \\ &+& \sum_{c=0}^{T} S(a-1,b-1,c,v) B(c,d,a+k), \end{array}$$

$$2 \leq a \leq n-k,\ 1 \leq b \leq a-1,$$

$$S(a,a,d,v) = \sum_{c=0}^{T} S(a-1,a-1,c,v)B(c,d,a+k),$$
$$2 \leq a \leq n-k.$$

Thus, the probability $H(k,n,v,w)$ to lose v out of k original multimedia packets and w out of $n-k$ FEC packets is

$$H(k,n,v,w) = \sum_{d=0}^{T} S(n-k,w,d,v). \quad (2)$$

The average number of lost multimedia packets E_{loss} equals

$$E_{loss} = 1 - \sum_{v=0}^{k} \sum_{w=0}^{n-k} H(k,n,v,w)R(v,w), \quad (3)$$

where $R(v,w) = \left\{ \begin{array}{ll} k, & v+w \leq n-k, \\ k-v, & else. \end{array} \right\}$.

V. Approximation Algorithms of low complexity to calculate E_{rec}

A. Complexity of the algorithm

In this section, we examine the computational complexity to calculate E_{loss} as defined in (3) and compare it with the latency requirements of a multimedia real-time application.

The calculation of E_{loss} first requires mk matrix multiplications and a singular value decomposition of a $(T+1) \times (T+1)$ matrix. These calculations have a complexity of $O(mkT^3)$. The calculation of the probabilities $T(a,b,d)$ and $S(a,b,c,d)$ requires $O(k^2T)$ and $O((n-k)^2kT^2)$ steps, respectively. Thus, the overall complexity of the *exact* algorithm is $\left(mkT^3 + k^2T + ((n-k)^2kT^2\right)$. Table I shows how long it took to calculate E_{loss} using a C-implementation on a 3.4 Gigahertz pentium PC for different queue lengths. As an example for the real time requirements of a typical multimedia application, we note that the payload of a Dolby AC-3 audio packet at a sample rate of 48 kbps is equal to 32 ms of audio content [3]. In a realtime scenario, the exact algorithm calculates E_{loss} for different values of n and k in order to find a parameter setting (n,k) that minimizes the multimedia packet loss after recovery. The execution time of the algorithm and the latency requirements of the Dolby AC-3 codec show that the adaption of the FEC parameters (n,k) based on the result delivered by the exact algorithm can happen only several tens of audio packets after the algorithm has received its input parameters p_A, p_C and p_D. Thus, the algorithm is too slow to dynamically adapt the FEC parameters.

B. Collapsed state model and approximation algorithm

In this section, we define an approximation algorithm to calculate E_{loss} in formula (3) in realtime. An adaptive FEC scheme will only apply FEC codes when packets are lost due to buffer overflow. The AP queue overflows when the accumulative arrival rate of both competing and multimedia traffic at the AP is higher than the departure rate from the AP. In such a situation, one expects that the AP queue is always full or nearly full, i.e., one expects that the entries S_k of the steady state vector S are nearly zero for all but a few values of k which are very close to T.

TABLE I
EXECUTION TIME OF THE ALGORITHM

Queue size T	Time in ms
400	1558
300	623
200	155
100	15
50	< 1

Our numerical experiments confirm these expectations. The fig. 4 shows the distributions of the steady state probabilities for different competing traffic rates p_C. We simulated CBR traffic with a sending rate $m=4$ and set $n=6$, $k=5$, $p_D=0.8$. We note that the sum of the arrival rates p_A+p_C is equal to p_D for $p_C=0.5$. For $p_C=0.52$, i.e., p_C is slightly higher than the equilibrium rate $p_C=0.5$, the steady state probabilities S_k rise sharply with increasing k, however steady state probabilities larger than 0.01 can be found for $k>95$. When increasing the competing traffic rate p_C to 0.6 such that the accumulative arrival rate p_A+p_C is significantly larger than the departure rate p_D, we find that the steady state probabilities S_k are all close to zero unless k is very close to T.

Any state with steady state probability $S_k=0$ has no significant impact on the dynamics of the steady state model as the system is virtually never in this state. Consequently, one can expect that these states can be neglected entirely for the calculation of E_{loss} without changing the result significantly.

Based on the above observation, we now propose an $approximation$ algorithm to calculate E_{loss} that only takes into account the steady states for which $S_k>\epsilon$, where ϵ is a pre-defined, small threshold value. We assume that $S_k \leq \epsilon$ for $k \leq T_0$, and we consider a "collapsed steady space vector" S^c with $T-T_0+1$ spaces: We collapse all states for which $k \leq T_0$ into the state S_0^c and consider the states for which $k>T_0$, individually. In the collapsed steady state model, we rename these S_k as $S_{k-T_0}^c$, $T_0 \leq k \leq T$.

For $0 \leq a,b \leq T-T_0$, we define the transition probability between the states S_a^c and S_b^c as $P_{a+T_0,b+T_0}^t$. We note that according to the model in section II, the transition matrix $P_{i+T_0,j+T_0}^t$ exactly describes the dynamics of the AP queue occupancy for the collapsed steady state model S^c if $i,j>0$, whereas it only approximately describes them if either $i=0$ or $j=0$. However, as we assume that the collapsed state S_0^c only occurs with a probability $\leq \epsilon T_0$, we anticipate that for a sufficiently small ϵ the numerical inaccuracy due to the usage of only approximatively correct transition probabilities to and from the state S_0^c will not significantly influence the calculation of the values of S_k^c for $1 \leq k \leq T-T_0$. Specifically, we expect that the absolute difference $|S_k^c - S_{k+T_0}|$ is very small for $1 \leq k \leq T-T_0$.

We calculate E_{loss} as in section IV using S^c instead of S and $T-T_0$ instead of T. This leads to a reduced computational complexity of $O(mk(T-T_0)^3)$. Table I shows that for T_0 close to T the approximation algorithm satisfies the latency requirements of a realtime application such as Dolby AC-3.

In practice, the algorithm must first determine the value T_0 in order to calculate the S^c. Our experiments described in the next section have shown that whenever $p_A + p_C > p_D + 0.1$, T_0 can be chosen as small as $T_0 = 10$.

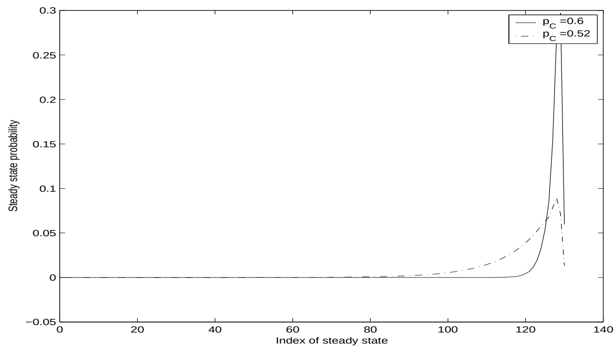

Fig. 4. Distribution of steady states for T=130, p_D=0.8, m=4, n=6, k=5

VI. Numerical results

In this section, we present network simulation results to verify the accuracy of the exact and approximation algorithms to calculate E_{loss}. We used the C programming language to build a model of the network and simulated 10.000.000 consecutive time slots.

Fig. 5 shows the change of the multimedia packet loss rate after FEC recovery as a function of the FEC parameter k for a given $n = 14$. The difference between the analytical and the simulation results is within the expected statistical variance of the simulation results and is so fine that the respective values cannot be distinguished in a graph.

Fig. 6 compares the exact algorithm with the results of the approximation algorithm. We choose $T = 100$ and $T - T_0 = 10$ and 20, respectively. We note that as $p_D = 0.8$ and $\frac{k}{nm} = 0.44$, the accumulative arrival rate equals the departure rate for $p_C = 0.355$. We conclude from fig. 6 that for $p_A + p_C > p_D$, the approximation delivers results that are numerically very close to the results delivered by the exact algorithm. For $p_A + p_C \leq p_D$, we see that the two algorithms give very different results. This is due to the fact that the approximation algorithm is based on the assumption that the queue length is nearly always in states S_k with k close to T. As this is only true in the range $p_A + p_C > p_D$, the approximation algorithm only approximates the exact algorithm in this range. For our application, this behavior of the approximation algorithm is sufficient as FEC protection is only required if $p_A + p_C > p_D$.

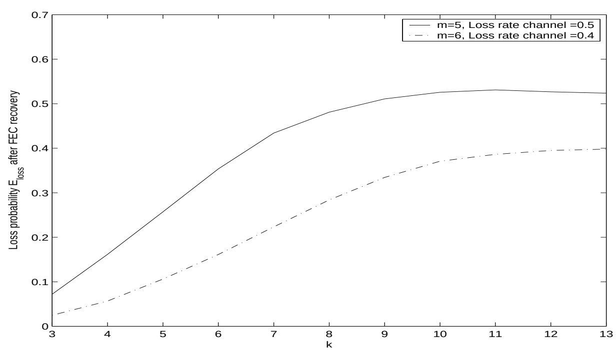

Fig. 5. The loss probability E_{loss} using FEC for changing k with T=130, p_C=0.72, n=15.

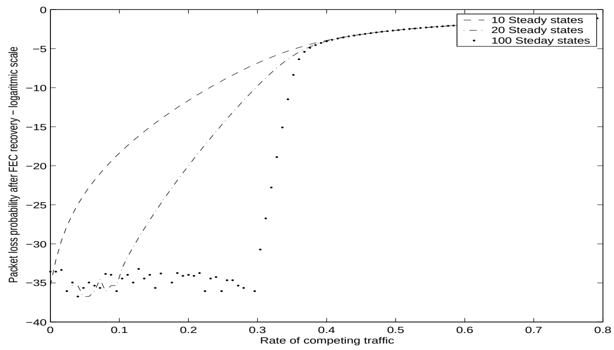

Fig. 6. Comparison of exact algorithm for $T = 100$ with the approximation algorithm for $T = 10, 20$ with p_D=0.8, m=3, n=4, k=3

VII. Conclusions

This paper presents algorithms to determine the optimal choice of FEC parameters (n, k) to protect a constant rate multimedia stream sent over a wireless network from packet losses due to packet dropping at the queue of the wireless AP. We develop a mathematical model of the wireless network that takes into account the exact arrival pattern of the multimedia stream, the accumulative arrival rate of all other traffic arriving at the wireless base station, the loss model of the wireless channel, and the queue size of the buffer of the wireless AP. We solve the underlying Markov model of the queue occupancy and develop an iterative procedure to calculate the effectiveness of the FEC parameters (n, k).

In order to apply this mathematical framework in practice, we propose an approximation algorithm of low complexity that allows to determine the optimal parameter settings of (n, k) for CBR multimedia traffic under real-time constraints. The accuracy of the algorithms is verified via network simulations.

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