Joint Optimization of Scale Factors and Huffman Code Books for MPEG-4 AAC

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Abstract—**The MPEG-4 AAC audio encoder achieves low bit rates by appropriately choosing the two encoding parameters Huffman Code Books and Scale Factors. Existing AAC implementations solve the implied optimization problem using an heuristic Two Loop Search. This paper presents two novel optimization techniques that for the first time find optimal solutions for the AAC optimization problem.**

I. Introduction

The future development and success of wireless and wired internet technologies depends on their ability to deliver multimedia content with an acceptable quality of service. Low bit rate coders such as MPEG-4 Advanced Audio Coding (AAC) play a crucial role in achieving this goal.

The AAC coder achieves transparent perceptual qualities at low bitrates by exploiting perceptual redundancies in the signal. For this purpose, the AAC encoder partitions each audio frame into a number of bands and for each band dynamically allocates bits for the transform coefficients. The transform coefficients and the side information are transmitted to the decoder. The main side information for AAC are the Scale Factors (SF) and the Huffman Code Books (HCB). These parameters are encoded differentially, such that the bit cost depends on the relation of the parameters at adjacent bands. To achieve a low bit rate encoding the encoder chooses the SFs and the $HCBs$ such that the total bit cost is below a given target rate, while ensuring that a predefined objective measure for the perceptual quality of the decoded signal is satisfied. The inter-band relationship of these parameters shows that their appropriate choice requires the solution of a complex optimization problem.

Two common perceptual quality measures are the average noise to mask ratio (ANMR) and the maximum noise to mask ratio (MNMR). The standard MPEG-4 AAC procedure to solve this optimization problem is the Two Loop Search (TLS) ([3]). The TLS uses an heuristic approach that neglects the inter-band dependencies of the side information and thus simplifies the problem significantly by optimizing the parameters *independently* for each band. In [1], [2], Aggarwal proposed a *joint* optimization of the encoding parameters. The problem is formalized as the minimization of the ANMR/MNMR subject to a constraint on the total transmission bit cost. Using a Lagrangian multiplier-based approach that penalizes any violation of the target rate, the ANMR problem is modeled as a trellis search, where each path through the trellis corresponds to a specific choice of the SF and HCB parameters. This method does not guarantee an optimal solution of the joint optimization problem. For the MNMR problem, an iterative Viterbi search is used, which always provides an optimal solution. In [7], the authors propose a less complex version of the methods in [1], that perform nearly as well as the original methods in [1].

The main contributions of this paper are two novel optimization techniques that solve the ANMR problem optimally. We also show a new optimal solution method for the MNMR problem. First, we develop a linear programming technique that applies to both quality measures. Our second method uses a dynamic programming technique that does not use a Lagrangian multiplier. This method applies to the ANMR problem only. To derive our optimization procedures, we give a mathematical model of the MPEG-4 AAC encoder that improves on the model used in [1].

Both the Trellis/Viterbi searches for the $ANMR$ and $MNMR$ problem in [2] as well as the dynamic programming based method proposed for the $ANMR$ problem in this paper are based on different applications of dynamic programming. To avoid confusion, we denote the methods in [2] as Trellis Searches, whereas our method is called the Dynamic Programming Method.

In the next section, we develop an analytic formulation of the problem under consideration. In sections III and IV, we present two novel, optimal techniques to the problem of determining the encoding parameters. In section V, we present numerical results, and we conclude in section VI.

II. Problem definition

The AAC encoder converts the time domain signal into the spectral domain using the modified discrete cosine transform. The 1024 spectral coefficients are grouped into N scale factor bands SFB_i, $1 \leq i \leq N$. Within the i-th band SFB_i, all coefficients are quantized using the same scalar quantizer step size, which is controlled by the i-th scale factor s_i selected from a range of M_1 possible values, i.e., $1 \leq s_i \leq M_1$. Also within SFB_i, the quantized coefficients are entropy encoded using a Huffman Code Book h_i selected from a set of M_2 possible values, i.e., $1 \leq h_i \leq M_2$,. We set $S = \{s_1, .., s_N\}$ and $H = \{h_1, .., h_N\}$. Both the s_i and h_i are indexes into sets of pre-determined Scale Factors and Huffman Code Books, respectively.
The noise to mask ratio (NMR) ([5]), which is the ratio of the quantization noise to the masking threshold ([8]), is the most widely used objective measure for the evaluation of an audio signal. Two types of NMR are widely used ([6]): the Average NMR ($ANMR$) and the Maximum NMR ($MNMR$). To define both NMR types analytically, we introduce the following notations: We define $d(s_i)$ as the quantization error of the i-th scale factor band if the i-th scale factor is chosen equal to s_i. w_i denotes the weighting applied to the i-th scale factor band which is defined as the inverse of the masking threshold (see [8]) of the i-th band. The ANMR is then analytically expressed as the sum

$$ANMR(S) : \;=\; \frac{1}{N}\sum_{i=1}^{N} w_i d(s_i). \tag{1}$$

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Using the same notation, the MNMR is expressed as

$$MNMR(S): \quad = \quad \max_{1\le i\le N} w_i d(s_i). \tag{2}$$

We now derive an analytic expression for the transmission rate. The transmission rate consists of three parts:

- $Q_i(s_i,h_i)$ counts the bits required to encode the quantized coefficients of the ith band with the SF value chosen as s_i and the HCB value chosen as h_i. According to the MPEG-4 AAC specification [4], $Q_i(s_i,1)=0$ for all possible values of s_i. We note that the function $Q_i(s_i,h_i)$ is also a function of the actual signal X., i.e., $Q_i(s_i,h_i) := Q_{X,i}(s_i,h_i)$. In general, for any two different signals X and Y, $Q_{X,i}(a,b)\neq Q_{Y,i}(a,b)$. In the sequel, we consider a fixed signal only and therefore omit the index X.
- $F(s_{i-1},s_i)$ counts the number of bits required to specify the SF for the ith band. As the SF are encoded differentially, we write $F(s_{i-1},s_i) := F(s_{i-1}-s_i)$.
- $G(h_{i-1},h_i) := G(h_i - h_{i-1})$ counts the number of bits needed to encode the HCB value of the i-th band.

We define the transmission rate $R(S,H)$ as follows:

$$R(S,H) = Q_1(s_1,h_1) + \sum_{i=2}^{N}\Big(Q_i(s_i,h_i) + F(s_{i-1}-s_i) + G(h_{i-1}-h_i)\Big). \tag{3}$$

The MPEG-4 AAC specification [4] mandates that the encoder cannot choose the same HCB value at more than 30 successive $SFBs$. We define the characteristic function $\sigma(i,b)$, where the argument i represents the i-th SFB, as $\sigma(i,b)=\left\{\begin{array}{ll}1 & \text{if } h_i=b,\\ 0 & \text{else.}\end{array}\right\}$. Now, we write the constraint on the choice of the vector H as

$$\sum_{i}^{i+30}\sigma(i,b) \quad \le \quad 30, \tag{4}$$

$1\le i\le N-30$, $1\le b\le M_2$. This constraint has not been considered in [1]. For a given rate threshold R_t and a distortion measure $NMR(S)\in\{ANMR(S), MNMR(S)\}$, the joint optimization problem is then defined as follows:

$$\begin{array}{lll} Minimize & NMR(S) & (5)\\ such\,that & R(S,H)\le R_t, & (6)\\ & (4)\, is\, satisfied. & \end{array}$$

III. An optimal solution based on mixed integer linear programming (MILP)

A. MILP formulation for the ANMR problem

We introduce the variable A_{i,d_1} that is set equal to 1 if $s_i - s_{i+1} = d_1 - M_1$, and is set equal to zero otherwise. Similarly, the variables B_{i,d_2} describe the difference of the values h_i and h_{i+1}. Thus, we define

$$A_{i,a} \quad = \quad \left\{\begin{array}{ll} 1, & \text{if } s_{i+1}-s_1 = a - M_1,\\ 0, & \text{else,}\end{array}\right\}$$
$$\forall i,\ 1\le i\le N-1,\ 1\le a\le 2M_1-1,$$
$$B_{i,b} \quad = \quad \left\{\begin{array}{ll} 1, & \text{if } h_{i+1}-h_i = b - M_2,\\ 0, & \text{else,}\end{array}\right\}$$
$$\forall i,\ 1\le i\le N-1,\ 1\le b\le 2M_2-1.$$

We define the variables $Z_{i,a,b}$ $\forall i$, $1\le i\le N$, $\forall a$, $1\le a\le M_1$ $\forall b$, $1\le b\le M_2$ that describe which values are taken by the variables s_i and h_i at the i-th stage:

$$Z_{i,a,b} = \left\{\begin{array}{ll} 1, & \text{if } s_i = a \bigvee h_i = b,\\ 0, & \text{else.}\end{array}\right\}$$

Finally, we define the variable $e_{i,m}$ as the weighted distortion that occurs at the i-th band if $s_i = m$, i.e., $e_{i,m} = w_i d(s_i = m)$, $\forall 1\le i\le N$, $1\le m\le M_1$. We model the ANMR joint optimization problem as follows:

$$\begin{array}{lll} Minimize & & \\ ANMR(Z): & = & \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\sum_{a=1}^{M_1}\sum_{b=1}^{M_2} e_{i,a}Z_{i,a,b} \qquad (7)\\ such\,that & & (8)\, and\,(12)-(17)\, are\, satisfied: \end{array}$$

$$R(Z,A,B) \quad \le R_t, \tag{8}$$

where

$$R(Z,A,B) = \sum_{i=1}^{N}\sum_{a=1}^{M_1}\sum_{b=1}^{M_2} Z_{i,a,b}Q_i(a,b) \tag{9}$$
$$+ \quad \sum_{i=1}^{N-1}\sum_{a=0}^{2M_1} A_{i,a}F^*(a) + \sum_{i=1}^{N-1}\sum_{b=0}^{2M_2} B_{i,b}G^*(b),$$

$$F^*(a_2-a_1) \quad = \quad F(M_1+a_2-a_1), \tag{10}$$
$$G^*(b_2-b_1) \quad = \quad G(M_2+b_2-b_1), \tag{11}$$
$$\sum_{a=1}^{2M_1-1} A_{i,a} \quad = \quad 1, \quad \forall i,\ 1\le 1\le N-1, \tag{12}$$
$$\sum_{b=1}^{2M_2-1} B_{i,b} \quad = \quad 1, \quad \forall i,\ 1\le 1\le N-1, \tag{13}$$
$$\sum_{a=1}^{M_1}\sum_{b=1}^{M_2} Z_{i,a,b} \quad = \quad 1, \quad \forall i,\ 1\le 1\le N, \tag{14}$$

$$A_{i,a} \ge \sum_{d_3=1}^{M_2} Z_{i,c,d_3} + \sum_{d_4=1}^{M_2} Z_{i+1,c+a-M_1,d_4} - 1, \tag{15}$$
$$B_{i,b} \ge \sum_{d_5=1}^{M_1} Z_{i,d_5,d} + \sum_{d_6=1}^{M_1} Z_{i+1,d_6,d+b-M_2} - 1, \tag{16}$$

for $1\le i\le N-1$, $1\le a\le 2M_1-1$, $1\le b\le 2M_2-1$, $\max(1,1+M_1-a)\le c\le \min(M_1,2M_1-a)$, $\max(1,1+M_2-b)\le d\le \min(M_2,2M_2-b)$.

$$30 \quad \ge \quad \sum_{i}^{i+30}\sum_{a=1}^{M_1} Z_{i,a,b}, \tag{17}$$

$\forall i$, $1 \le i \le N-30$, $\forall b$, $1 \le b \le M_2$. The variables $Z_{i,a,b}$ are required to be non-negative integers and the variables $A_{i,a}$ and $B_{i,b}$ are required to be non-negative real numbers.

We first show that the variables $Z_{i,a,b}$, $A_{i,a}$, and $B_{i,b}$ consistently express the choice of the SF and HCB values at all $SFBs$. The equation (14) ensures that for each band i, exactly one variable $Z_{i,a,b}$ is equal to 1, whereas all the other variables are equal to zero, i.e., exactly one SF and one HCB are chosen per band. For fixed values of the variables $Z_{i,a,b}$, the equations (12) and (15) ensure that if $s_{i+1} - s_1 = a - M_1$, then $A_{i,a} = 1$ and all other $A_{i,a} = 0$ as required by the definition of $A_{i,a}$. The equations (13) and (16) imply the analogous relation for the variables $B_{i,b}$.

By the definition of the variables $Z_{i,a,b}$, the condition (4) is modeled by the equation (17). Also, the definitions of the variables $Z_{i,a,b}$ and $e_{i,m}$, show that the definitions of the objective function in (7) and (1) are identical.

The functions $F^*(n)$ and $G^*(n)$ are identical to the function $F(n)$ and $G(n)$ with the argument shifted by M_1 and M_2, respectively. The shifts have been introduced in order to avoid negative variables in the MILP formulation. Thus, the expressions (3) and (9) are identical.

The MILP formulation is solved using the Simplex method and then applying a branch and bound method for the integer solution. The variables $A_{i,a}$ and $B_{i,b}$ are not explicitly required to be integer, but their integrity is achieved via the relations (12) and (15) and the relations (13) and (16), respectively. This decreases the complexity for the branch and bound algorithm.

B. MILP formulation for the MNMR problem

We define the variable p as the maximum weighted noise to mask ratio of all bands, i.e., $p = \max_{1 \le i \le N} w_i d(s_i)$. The MNMR problem is modeled as follows:

$$\begin{aligned} &Minimize \quad p \quad , \\ &such\,that \quad (8)\,and\,(12)-(18)\,are\,satisfied: \\ &\sum_{a=1}^{M_1}\sum_{b=1}^{M_2} e_{i,a} Z_{i,a,b} \le p \quad , \quad \forall i,\, 1 \le i \le N. \end{aligned} \tag{18}$$

IV. An optimal solution based on dynamic programming

In this section, we present an optimal solution algorithm for the $ANMR$ problem as defined in (5) that is based on a dynamic programming technique.

Similar to [1], we construct a trellis as follows: The i-th stage of the Trellis corresponds to SFB_i. The states in each stage correspond to all possible combinations of the SF and HCB variables s_i and h_i. We connect all stages of SFB_{i-1} with all states of SFB_i. Each path through the trellis corresponds to a choice of the encoding parameters s_i and h_i. We call the sum of the weighted distortions of the states that form a path (see (1)) divided by the number of $SFBs$ $-N-$ the distortion associated with the path. We define the cost to proceed from the state (s_{i-1}, h_{i-1}) in SFB_{i-1} to state (s_i, h_i) in SFB_i as $Q_i(s_i, h_i) + F(s_{i-1} - s_i) + G(h_{i-1} - h_i)$. Defining the cost for choosing the state (s_1, h_1) in the first stage SFB_1 as $Q_1(s_1, h_1)$, we see that the cost of the path $(s_1, h_1), ..., (s_N, h_N)$ through the trellis is $Q_1(s_1, h_1) + \sum_{i=2}^{N} Q_i(s_i, h_i) + F^*(s_{i-1} - s_i) + G^*(h_{i-1} - h_i)$. This expression is identical to the expression for the transmission rate in (9). Thus, the solution of the problem (5) or (7) - (17) is equivalent to finding a path through the trellis that minimizes the distortion associated with this path under the constraint that the cost of the path is below a given threshold R_t. As the solution algorithm described below is based on dynamic programming, the non-local condition (4) is not taken into account. We now define an algorithm to determine the cheapest path through the trellis:

1. For each integer y, $0 \le y \le R_t$, and each state $(s_2, h_2) \in SFB_2$, we find a path from SFB_1 to SFB_2, the cost of which is defined as $f_2^{(s_2,h_2)}(y)$. It is defined as

$$f_2^{(s_2,h_2)}(y) \quad = \quad w_2 d(s_2) + \min_{(s_1,h_1)\in SFB_1} w_1 d(s_1)$$

where the minimization is over all (s_1, h_1) such that

$$Q_1(s_1, h_1) + Q_2(s_2, h_2) + F^*(s_1 - s_2) + G^*(h_1 - h_2) \le y. \tag{19}$$

2. For the i-th band SFB_i, $3 \le i \le N$, do the following: For each point $(s_i, h_i) \in SFB_i$ and each integer y, $0 \le y \le R_t$, set

$$\begin{aligned} &f_i^{(s_i,h_i)}(y) = w_i d(s_i) + \\ &\min_{(s_{i-1},h_{i-1})\in SFB_{i-1}} f_{i-1}^{(s_{i-1},h_{i-1})}(y - S(s_{i-1}, h_{i-1}, s_i, h_i)), \end{aligned}$$

where

$$\begin{aligned} &S(s_{i-1}, h_{i-1}, s_i, h_i) \\ = \quad &Q_i(s_i, h_i) + F^*(s_{i-1} - s_i) + G^*(h_{i-1} - h_i). \end{aligned}$$

3. $\min_{(s_i,h_i)\in SFB_N} \in f_N^{(s_i,h_i)}(R_t)$ is the cost of the cheapest path through the trellis.

Remarks: In step 1, $f_2^{(s_2,h_2)}(y)$ does not exist if the inequality (19) cannot be satisfied for any pair of states (s_1, h_1) and (s_2, h_2). If in step 2, for a certain state $(s_{i-1}^*, h_{i-1}^*) \in SFB_{i-1}$, $f_{i-1}^{(s_{i-1}^*,h_{i-1}^*)}(y - S(s_{i-1}^*, h_{i-1}^*, s_i, h_i))$ does not exist due to the reason given above (for $i = 2$) or because $y - S(s_{i-1}^*, h_{i-1}^*, s_i, h_i) < 0$ (for $i > 2$), then the minimization over (s_{i-1}, h_{i-1}) excludes (s_{i-1}^*, h_{i-1}^*).

V. Numerical comparison of different optimization techniques

In this section, we use the new methods proposed in this paper to evaluate the performance of the previous approaches, Two Loop Search and the Trellis Search. We compared the different methods using 4600 audio frames and assumed 49 $SFBs$, 60 SFs and 12 $HCBs$.

For the $ANMR$ problem, both the MILP solution and the Dynamic Programming Method provide an optimal solution. Only the MILP based approach takes into account

TABLE I

COMPARISON OF AVERAGE $ANMR$ VALUES

Compared optimization techniques	ANMR ratio
Two Loop Search/Dynamic Prog.	2.4
Trellis Search/Dynamic Prog.	1.1
Two Loop Search/Trellis Search	2.2

TABLE II

COMPARISON OF AVERAGE $MNMR$ VALUES

Compared optimization techniques	MNMR ratio
Two Loop Search/Trellis Search	2.2

the condition (4). However, our numerical experiments showed that the Dynamic Programming Method only very rarely determines sets of encoding parameters that contradict the condition (4). The Two Loop Search takes into account the condition (4), whereas the Trellis Search ([1]) does not. As the dynamic programming based solution, the Trellis Search very rarely violates the condition (4). Thus, in order to compare the different optimization techniques, we compared the performance of the Dynamic Programming Method, the Trellis Search, and an implementation of the Two Loop Search by Dolby Laboratories, and did not consider the cases where the Trellis Search violated the condition (4).

For the ANMR problem, table I shows that the Trellis Search is on average about 10% worse than the optimal solution. The Two Loop Search is on average more than two times worse than both other methods. The CDFs of the ratio of the distortions achieved by the different optimization techniques are shown in figure 1. The solution of the MILP with an LP solver based on the Simplex method running on a Pentium 5, 1.7 Ghz took 70 seconds, whereas the Dynamic Programming Method needed about 12 minutes to solve the problem. The Trellis search needs about 2 seconds to solve the problem. The huge gap in running time and the small difference of 10 % in the average distortion show the strength of the Trellis Search. The Two Loop Search is the only real-time method as it needs several milliseconds to solve the ANMR problem.

For the $MNMR$ problem, both the Trellis search and the MILP presented in section VII.B provide optimal solutions. As for the $ANMR$ problem, we compare the Trellis Search with the Two Loop search, and disregard the cases where the Trellis Search violates the constraint (4). Table II shows that the Two Loop Search is on average more than two times worse than the Trellis Search. The corresponding CDF is shown in figure 2. The running times for the Trellis Search is about 2 seconds and the Two Loop Search needs several milliseconds.

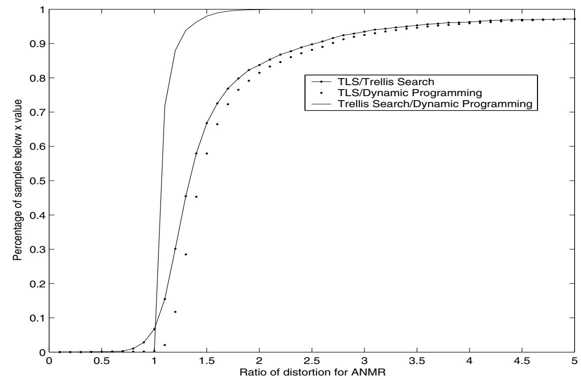

Fig. 1. Comparison of TLS, Trellis Search and Dynamic Programming for ANMR

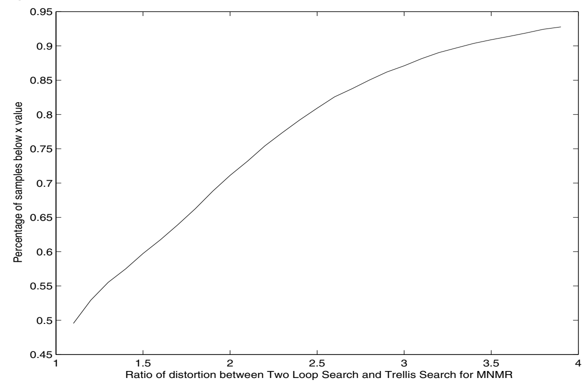

Fig. 2. Comparison of Two Loop search and Trellis search for MNMR

VI. CONCLUSIONS

This paper proposes new methods to solve the joint optimization problem for the MPEG-4 AAC encoder. We developed an analytic model of the encoding process that improves on previous approaches. Then, we presented two methods that for the first time solve the joint optimization problem optimally. The first method models the joint optimization problem as a Mixed Integer Linear program. The second method uses a Dynamic Programming approach.

For the $ANMR$ problem, the Trellis Search achieves distortions that are about 10% worse than the optimal solutions. However, the optimal MILP and the Dynamic Programming Method are computationally far more complex than the Trellis Search. For both the $ANMR$ and the $MNMR$ problem, the Two Loop Search is the only real-time capable solution, but it achieves distortions that are significantly higher than the optimal distortions.

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